

Birzeit University  
Mathematics Department  
Math 234

(97)

First Hour Exam

First semester 2013/2014

Student name: ~~Handwritten name~~

"5"

Student no: ~~Handwritten number~~

Question #1 (24%): Which of the following statements is true and which is false:  
**False 1-** If  $A$  is an  $n \times n$  matrix and the system  $AX=0$  has a nontrivial solution then  $A$  is nonsingular

**True 2-** If  $A$  and  $B$  are  $n \times n$  matrices and  $AB$  is nonsingular then both  $A$  and  $B$  are nonsingular.

**False 3-** If  $A$  is  $4 \times 4$  matrix then  $|-A| = -|A|$

**False 4-** If  $A$  is  $3 \times 3$  matrix and  $A = -A^T$  then  $A$  is singular

$N = 0$   
 $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{vmatrix} = 2 \neq 0$   
 $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{vmatrix} = 2 \neq 0$

**False 5-** The product of two symmetric non matrices is symmetric.

**True 6-** If  $A$  is a nonsingular matrix then the matrix  $A^T$  is nonsingular also.

**False 7-** Any non homogenous system of linear equations that has a nontrivial solution must have infinite number of solutions.

**False 8-** If  $A, B, C$  are  $2 \times 2$  matrices with  $AB=AC$  then  $B=C$ .

**False 9-** If  $A$  and  $B$  are  $2 \times 2$  matrices such that  $A \cdot B = 0$  then  $A=0$  or  $B=0$ .

**True 10-** If  $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 5 & 0 & 1 \end{pmatrix}$  then  $A^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -5 & 0 & 1 \end{pmatrix}$

**True 11-** If  $A, B$  are  $3 \times 3$  matrices with  $|A|=4, |B|=5$  then  $|2A^{-1}B|=10$

**True 12-** If  $A = \begin{pmatrix} 2 & 5 & 7 \\ 1 & 3 & 4 \\ 2 & 1 & 6 \end{pmatrix}$  then the  $(2,3)$  entry of  $A^{-1}$  is  $\frac{1}{3}$

$A^{-1}_{32} = \frac{A_{32}}{|A|} = \frac{1}{3}$

$\begin{bmatrix} 2 & 5 & 7 \\ 1 & 3 & 4 \\ 2 & 1 & 6 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 1 & 3 & 4 \\ 2 & 5 & 7 \\ 2 & 1 & 6 \end{bmatrix}$   
 $\xrightarrow{R_2 - 2R_1, R_3 - 2R_1} \begin{bmatrix} 1 & 3 & 4 \\ 0 & -1 & -1 \\ 0 & -5 & -2 \end{bmatrix}$   
 $\xrightarrow{R_2 \cdot (-1)} \begin{bmatrix} 1 & 3 & 4 \\ 0 & 1 & 1 \\ 0 & -5 & -2 \end{bmatrix}$   
 $\xrightarrow{R_3 + 5R_2} \begin{bmatrix} 1 & 3 & 4 \\ 0 & 1 & 1 \\ 0 & 0 & 3 \end{bmatrix}$   
 $\xrightarrow{R_3 \cdot \frac{1}{3}} \begin{bmatrix} 1 & 3 & 4 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$   
 $\xrightarrow{R_2 - R_3} \begin{bmatrix} 1 & 3 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$   
 $\xrightarrow{R_1 - 3R_2} \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$   
 $\xrightarrow{R_1 - 4R_3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = A^{-1}$

$A^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -5 & 0 & 1 \end{pmatrix}$   
 $A^{-1}_{32} = -5$

13- If the coefficient matrix of the system  $AX=b$  is  $A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & 1 \\ 0 & 4 & 1 \end{pmatrix}$

True

Then the system must have a unique solution

14- Any nonsingular matrix can be written as a product of elementary matrices.

15- The product of two elementary matrices is elementary

16-  $|AB| = |BA|$  for any two  $n \times n$  matrices A and B  
 $|A| |B| = |B| |A|$

Question #2(30%): Circle the correct answer:

1- If  $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ , and  $|A| = 6$ , and  $B = \begin{bmatrix} 2a & 2b & 2c \\ 4g & 4h & 4i \\ -d & -e & -f \end{bmatrix}$  then  $|B| =$   
 $-1 \times 2 \times -1 \times 4 = 8 \quad |A| = 86 = 48$

- a) 48
- c) -24

- b) 48
- d) 24

2- If A, B are two  $n \times n$  matrices then

- a)  $\det(AB^T) = \det(AB) \rightarrow |A| |B^T| = |A| |B|$  ✓
- b)  $\det(\alpha A) = \alpha \det(A) \times \alpha^n |A|$
- c)  $\det(A+B) = \det(A) + \det(B)$  ✗

3- If  $AX=b$  has no solution where A is an  $n \times n$  matrix and b is an  $n \times 1$  matrix then:

- a- A is row equivalent to  $I_n$
- b- A is nonsingular.
- c- A is a product of elementary matrices.
- d-  $AX=0$  has infinitely many solutions.

4- The conditions on a, b such that the system  $\begin{cases} ax + y = 1 \\ 2x + y = b \end{cases}$

has infinite number of solution is

- a)  $a=2$  and  $b=1$
- b)  $a \neq 2$  and  $b=1$
- c)  $a=2$  and  $b \neq 1$
- d)  $a \neq 2$  and  $b \neq 1$

$$\begin{bmatrix} a & 1 & 1 \\ 2 & 1 & b \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 1 & b \\ a & 1 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1/2 & b/2 \\ 0 & 1 - \frac{a}{2} & 1 - \frac{a}{2} \end{bmatrix}$$

5- If A and B are  $n \times n$  nonsingular matrices then:

- a-  $(AB)^T = A^T B^T$
- b-  $(A+B)^T = A^T + B^T$
- c-  $(AB)^{-1} = A^{-1} B^{-1}$
- d-  $|\alpha A| = \alpha^n |A|$

$$\left. \begin{aligned} 1 - \frac{a}{2} &= 0 & 1 - \frac{a}{2} &= b \\ 1 - \frac{a}{2} & & 1 &= \frac{a}{2} \\ a &= 2 & 2 &= a \cdot b \end{aligned} \right\} \begin{aligned} a &= 2 \\ b &= 1 \end{aligned}$$

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Question 1 (39 points). Mark each of the following by True or False

- (1) (T) An  $n \times n$ -matrix  $A$  is nonsingular if and only if  $A$  is a product of elementary matrices.
- (2) (F) Any two  $n \times n$ -singular matrices are row equivalent.
- (3) (T) If  $A$  is a singular matrix and  $U$  is the row echelon form of  $A$ , then  $\det(U) = 0$ .
- (4) (T) If  $A$  is a  $3 \times 3$ -matrix and the system  $Ax = \begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix}$  has a unique solution, then the system  $Ax = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$  has only the zero solution.
- (5) (T) If an  $n \times n$ -matrix  $A$  is nonsingular, then Cramer's rule can be used to solve the system  $Ax = b$ .
- (6) (T) Let  $A$  be a  $4 \times 3$ -matrix with  $a_2 = a_3$ . If  $b = a_1 + a_2 + a_3$ , where  $a_j$  is the  $j$ th column of  $A$ , then the system  $Ax = b$  will have infinitely many solutions.
- (7) (T) If  $y, z$  are solutions to the system  $Ax = 0$ , then any linear combination of  $y, z$  is also a solution to  $Ax = 0$ .
- (8) (T) If  $A$  is a singular and  $B$  a nonsingular  $n \times n$ -matrices, then  $AB$  is singular.
- (9) (F) If a matrix  $B$  is obtained from  $A$  by multiplying a row of  $A$  by a real number  $c$ , then  $|A| = c|B|$ .
- (10) (T) If  $A, B$  are  $n \times n$ -matrices,  $|A| = 3$  and  $|B| = -2$ , then  $|A^{-1}B^T| = \frac{2}{3}$ .
- (11) (T) If  $A$  is a singular matrix, then  $A \text{adj}(A) = 0$ .
- (12) (T)  $S = \left\{ \begin{pmatrix} a \\ b \\ 0 \end{pmatrix} : a, b \in \mathbb{R} \right\}$  is a subspace of  $\mathbb{R}^3$ .
- (13) (F) If  $A$  is row equivalent to  $B$ , then  $\det(A) = \det(B)$ .
- (14) (F) If  $A$  is a nonsingular  $n \times n$ -matrix and  $c$  is a nonzero real number, then  $(cA)^{-1} = \frac{1}{c}A^{-1}$ .
- (15) (F) If  $A$  is a singular  $n \times n$ -matrix,  $b \in \mathbb{R}^n$ , then the system  $Ax = b$  has infinitely many solutions.
- (16) (F) If  $A, B$  are  $4 \times 4$ -matrices and  $AB$  is the zero matrix, then  $\det(A) = 0$ .

~~det(A)~~

$$\frac{1}{\det(A)} \det(cA)$$

$$AB = 0$$

$$\det(AB) = \frac{\det(A)}{0} \frac{\det(B)}{0}$$

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